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SHRINKAGE ESTIMATION METHOD OF PARAMETER WEIBULL DISTRIBUTION

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ABSTRACT

Consider the Weibull distribution. The parameters of this distribution are estimated by the maximum likelihood method and Bayes method. In this study we present shrinkage estimator between maximum likelihood $\hat{\theta}_M$ and Bayes estimators $\hat{\theta}_B$ that make mean square error (MSE) less than other standard method. Using linear combination between maximum likelihood method and Bayes method to obtain a new estimator $\hat{\theta}$ then simulation study will used to compare between shrinkage estimator, Maximum likelihood estimator and Bayes estimator to find the best (less mean square error) with different sample size and Mathlab program.

KEYWORDS: Weibull Distribution, Shrinkage Estimator, Bayes Estimator, Maximum Likelihood Estimator, Simulation

1. INTRODUCTION

Chiou and Han studied shrinkage estimators of the parameter exponential distribution; they introduced the usual preliminary test estimators of the threshold parameter of the exponential distribution in censored samples. Morris, Baggerly and Coombes developed a new Bayesian estimation procedure that quantifies prior information about two characteristics, yielding a nonlinear shrinkage estimator with efficiency advantages over the MLE. In 1996 Singh and Raghuvanshi studied the problem of estimation of variance in exponential density when a prior point estimate is available. In 2008 Baklizi and Ahmed studied three classes of point estimators, namely, the unrestricted estimator, the shrinkage estimator and shrinkage preliminary test estimator and mean squared errors are derived and compared.

In this paper we studied shrinkage estimator, $\tilde{\theta}$ between Bayes and Maximum likelihood estimators. Using simulation study, we compared between estimators shrinkage estimator, Maximum likelihood estimator and Bayes estimator of parameter Weibull distribution to find the best depend on less mean square error MSE.

The estimators of parameter Weibull distribution was estimated (see Al Omari, Alkutubi and Akma2010), supposed $t_1,\ldots,t_n$ be a random sample of size n with distribution function and probability density function. Using Bayesian and Maximum likelihood (MLE) methods to obtain these estimators The probability density function of Weibullcase is $f(t;\theta,p) = \frac{p}{\theta} t^{p-1} \exp(-\frac{t^p}{\theta})$, and $\hat{\theta}_B = \frac{\sum_{i=1}^{n} t^p{}_i}{n-1}$ is the Bayes estimator. The mean square error of Bayes estimator is given by expected value and variance, such that $MSE(\hat{\theta}_B) = \frac{n+1}{(n-1)^2}\theta^2$, Where,

$E\left(\hat{\theta}_B\right)=\dfrac{n}{n-1}\theta$, $Var\left(\hat{\theta}_B\right)=\dfrac{n}{(n-1)^2}\theta^2$.

Also the maximum likelihood estimator of exponential distribution is given by $\hat{\theta}_M = \dfrac{\sum_{i=1}^{n} t^{p}{}_{i}}{n}$, And $MSE\ (\hat{\theta}_M) = \dfrac{\theta^2}{n}$, is the mean square error of maximum likelihood estimator.

2. MATERIALS AND METHODS

2.1 Shrinkage Estimator $\tilde{\theta}_1$ between Bayes and MLE

In this section, we obtain shrinkage estimator $\tilde{\theta}_1$ Weibull distribution from Bayesian and maximum likelihood estimators by linear such that, ,

$$\tilde{\theta}_1 = p_1\hat{\theta}_M + (1 - p_1)\hat{\theta}_B$$

The value of p_1 which minimizes $MSE(\tilde{\theta}_1)$ is, $p_1 = \dfrac{n^2 + 2n + 1}{4n^2 - n + 2}$

Then the shrinkage estimator $\tilde{\theta}_1$ between Bayes and MLE of Weibull distribution is given by

$$\tilde{\theta}_1 = (\frac{n^2 + 2n + 1}{4n^2 - n + 2})\hat{\theta}_M + (1 - (\frac{n^2 + 2n + 1}{4n^2 - n + 2}))\hat{\theta}_B$$

2.2 Shrinkage Estimator $\tilde{\theta}_{21}$ Between $\tilde{\theta}_1$ and MLE

We can obtain the shrinkage estimator $\tilde{\theta}_{21}$ for Weibull distribution from $\tilde{\theta}_1$ and MLE depending on linear combination to get the following equation,

$$\tilde{\theta}_{21} = p_{21}\tilde{\theta}_1 + (1 - p_{21})\hat{\theta}_M$$

To find the value of p_{21} which minimizes $MSE\ (\tilde{\theta}_{21})$, we follows

$$\tilde{\theta}_{21} - \theta = [p_{21}\tilde{\theta}_1 + (1 - p_{21})\hat{\theta}_M] - \theta$$

Taking expected valued,

$$E(\tilde{\theta}_{21} - \theta)^2 = p^2{}_{21}E(\tilde{\theta}_1 - \theta)^2 + (1 - p_{21})^2 E(\hat{\theta}_M - \theta)^2 + p_{21}(1 - p_{21})E(\tilde{\theta}_1 - \theta)(\hat{\theta}_M - \theta),$$

Then

$$MSE(\tilde{\theta}_{21}) = p^2{}_{21}MSE(\tilde{\theta}_1) + (1 - p_{21})^2 MSE(\hat{\theta}_M) + 2p_{21}(1 - p_{21})[E(\tilde{\theta}_1\hat{\theta}_M) - E(\tilde{\theta}_1\theta) - E(\hat{\theta}_M\theta) - E\theta^2],$$

Let $\frac{\partial MSE(\tilde{\theta}_{21})}{\partial P_{21}} = 0$, implies the value of p_{21} which minimizes $MSE(\tilde{\theta}_{21})$ is

$$p_{21} = \frac{MSE(\hat{\theta}_M) - E(\tilde{\theta}_1\hat{\theta}_M) + E(\tilde{\theta}_1\theta) + E(\hat{\theta}_M\theta) - E(\theta)^2}{MSE(\tilde{\theta}_1) + MSE(\hat{\theta}_M) - 2E(\tilde{\theta}_1\hat{\theta}_M) + 2E(\tilde{\theta}_1\theta) + 2E(\hat{\theta}_M\theta) - 2E(\theta^2)}$$, where

$$E(\tilde{\theta}_1) = \left(\frac{4n^6 - 6n^5 + n^4 + 2n^3 - 6n^2 - 1n}{n^6 - 10n^5 + 12n^4 - 18n^3 + 9n^2 - 3n}\right)\theta$$, $E(\hat{\theta}_M) = \theta$

$$MSE(\tilde{\theta}_1) = \left(\frac{8n^8 - 2n^7 - n^6 + n^5 - 6n^4 - 4n^3 + 6n^2 + 2n}{2n^8 - 29n^7 + 25n^6 - 24n^5 + 33n^4 - 10n^3 + 16n^2 - 4n + 2}\right)\theta^2$$ and

$MSE(\hat{\theta}_M) = \frac{\theta^2}{n}$, Then $p_{21} = \frac{x_{21}}{y_{21}}$, where

$$x_{21} = \left(\frac{4 - 2n + 16n^2 - 31n^3 + 36n^4 - 27n^5 + 26n^6 - 10n^7}{6n^7 - 14n^6 + 19n^5 - 12n^4 + 16n^3 - 12n^2 + 2n}\right)\theta^2$$
$$+ \left(\frac{\sum t_i\left(n - 2n^2 + 10n^3 - 12n^4 + 13n^5 - 12n^6 - n^7\right)}{14n^8 - 22n^7 + 27n^6 - 26n^5 + 20n^4 - 2n^3 + n^2}\right)\theta$$

and,

$$y_{21} = [(57n^{15} + 29n^{14} + 4540n^{13} - 361n^{12} - 670n^{11} + 2231n^{10} - 172n^9 + 142n^8 - 565n^7 - 66n^6 + 19n^5 - 211n^4 + 121n^3 - 40n^2 + 21n - 4) \div (2n - 10n^2 - 42n^3 - 130n^4 + 322n^5 - 652n^6 + 1342n^7 - 2431n^8 + 2784n^9 - 2349n^{10} + 2958n^{11} - 1226n^{12} + 306n^{13} - 135n^{14} + 27n^{15})]\theta^2 +$$
$$\left[\frac{\sum\left(-2n^7 - 6n^6 + 31n^5 - 29n^4 + 33n^3 - 4n^2 - 3n\right)}{5n^8 - 17n^7 + 13n^6 - 14n^5 + 12n^4 - 3n^3 + n^2}\right]\theta$$

Then the shrinkage estimator of Weibull distribution $\tilde{\theta}_{21}$ between $\tilde{\theta}_1$ and MLE is given by

$$\tilde{\theta}_{21} = \frac{x_{21}}{y_{21}}\tilde{\theta}_1 + (1 - \frac{x_{21}}{y_{21}})\hat{\theta}_M$$

2.3 Shrinkage Estimator $\tilde{\theta}_{22}$ Between $\tilde{\theta}_1$ and Bayes

In the same way in above, we can get the shrinkage estimator $\tilde{\theta}_{22}$ of Weibull distribution between $\tilde{\theta}_1$ and Bayes such that

$$\tilde{\theta}_{22} = p_{22}\tilde{\theta}_1 + (1 - p_{22})\hat{\theta}_B$$

The value of p_{22} which minimizes $MSE\ (\tilde{\theta}_{22})$ as follows

$$p_{22} = \frac{MSE\ (\hat{\theta}_B) - E(\tilde{\theta}_1\hat{\theta}_B) + E(\tilde{\theta}_1\theta) + E(\hat{\theta}_B\theta) - E(\theta^2)}{MSE\ (\tilde{\theta}_1) + MSE\ (\hat{\theta}_B) - 2E(\tilde{\theta}_1\hat{\theta}_B) + 2E(\hat{\theta}_B\theta) + 2E(\tilde{\theta}_1\theta) - 2E(\theta^2)}$$

Depending on $MSE(\hat{\theta}_B)$, $MSE(\tilde{\theta}_1)$, $E(\hat{\theta}_B)$ and $E(\tilde{\theta}_1)$ we can obtain p_{22}, such that

$p_{22} = \frac{x_{22}}{y_{22}} \times \frac{w_{22}}{z_{22}}$, Where x_{22}, y_{22}, w_{22} and z_{22} is given by

$$x_{22} = 60n^{16} - 570n^{15} + 2235n^{14} - 5642n^{13} + 9721n^{12} - 52310n^{11} + 16940n^{10} - 18421n^9 + 16321n^8 - 98542n^7 + 56542n^6 - 39642n^5 + 17890n^4 - 553n^3 + 167n^2 - 12n + 6$$

$$y_{22} = 12n^{16} - 123n^{15} + 743n^{14} - 2656n^{13} + 4643n^{12} - 6750n^{11} + 9378n^{10} - 13674n^9 + 9321n^8 - 7033n^7 + 4476n^6 - 2287n^5 + 926n^4 - 390n^3 + 77n^2 - 9n + 4$$

$$w_{22} = 130n^{25} - 1787n^{24} + 11652n^{23} - 58750n^{22} + 148702n^{21} - 338904n^{20} + 599433n^{19} - 587526n^{18} + 687532n^{17} - 1223842n^{16} + 534329n^{15} - 787456n^{14} + 590818n^{13} - 619083n^{12} + 523976n^{11} - 390395n^{10} + 254273n^9 - 213304n^8 + 56064n^7 - 25698n^6 + 106114n^5 - 6591n^4 + 1775n^3 - 179n^2 + 34n - 2$$

$$z_{22} = 18n^{24} - 169n^{23} + 944n^{22} - 3238n^{21} + 7754n^{20} - 15453n^{19} + 27895n^{18} - 28800n^{17} + 54302n^{16} - 45678n^{15} + 57898n^{14} - 56509n^{13} + 45748n^{12} - 45433n^{11} + 34592n^{10} - 37894n^9 + 24551n^8 - 15138n^7 + 8060n^6 - 3622n^5 + 1336n^4 - 387n^3 + 79n^2 - 9n + 1$$

Then the Shrinkage Estimator $\tilde{\theta}_{22}$ Between $\tilde{\theta}_1$ and Bayes is given by

$$\tilde{\theta}_{22} = \left(\frac{x_{22}}{y_{22}} \times \frac{w_{22}}{z_{22}}\right)\tilde{\theta}_1 + \left(1 - \left(\frac{x_{22}}{y_{22}} \times \frac{w_{22}}{z_{22}}\right)\right)\hat{\theta}_B$$

3 SIMULATION STUDY

In this study, we chooses samples sizes n=30, 60, 90, with parameter value θ = 0.5, 1, 1.5 and R=1000 of replication. Using mean square error (MSE) to compare between all estimators $\tilde{\theta}_1, \tilde{\theta}_{21}, \tilde{\theta}_{22}, \widehat{\theta_B}$ and $\widehat{\theta_M}$, where

$$MSE\ (\tilde{\theta}) = \frac{\sum_{i=1}^{R}\left(\hat{\theta}_i - \theta\right)^2}{R},$$

The simulation program is written by Matlab program. And the results are introduced and tabulated in Table 1 for the MSE of all estimators for all sample sizes and θ values respectively.

Table 1: MSE of Estimator Exponential Distribution

N	θ	$\tilde{\theta}_1$	$\tilde{\theta}_{21}$	$\tilde{\theta}_{22}$	$\widehat{\theta_B}$	$\widehat{\theta_M}$
30	0.5	0.0324	0.0320	0.0321	0.0312	0.0312
	1	0.0322	0.0318	0.0319	0.0321	0.0325
	1.5	0.0310	0.0307	0.0309	0.0309	0.0311
60	0.5	0.0226	0.0220	0.0221	0.0222	0.0222
	1	0.0218	0.0217	0.0218	0.0219	0.0219
	1.5	0.0217	0.0216	0.0217	0.0220	0.0221
90	0.5	0.0134	0.0126	0.0129	0.0131	0.0135
	1	0.0088	0.0085	0.0086	0.0087	0.0089
	1.5	0.0050	0.0047	0.0048	0.0048	0.0049

From this Table, the shrinkage estimator $\tilde{\theta}_{21}$ is the best shrinkage estimator from all estimators and for all sample size and parameter value.

4 CONCLUSIONS

The generator shrinkage estimator $\tilde{\theta}_{21}$ is the best estimator (less mean square error). The effects of sample size on the mean square error of all estimators refer to the MSE decreases as n increases.

REFERENCES

1. Alkutubi H. S. and Ibrahim N. A. 2009 [a]. Bayes estimator for exponential distribution with extension of Jeffery prior information. Malaysian Journal of Mathematical Sciences 3(2): 297- 313.
2. Alkutubi H. S. and Ibrahim N. A., 2009 [b], on the estimation of survival function and parameter exponential life time distribution. Journal of Mathematics and Statistic 5(2): 130-135.
3. Alkutubi H. S. and Ibrahim N. A., 2009 [c]. Comparison three Shrinkage estimators of parameter exponential distribution. International Journal of Applied Mathematics 22(5).
4. Alomari M, Alkutubi H and Akma N. Comparison of the Bayesian and maximum likelihood estimation for Weibull Distribution. Journal of Mathematics and statistics 6 (2): 100-104, 2010.
5. Baklizi A. and Ahmed S. E. 2008. On the estimation of reliability function in a Weibull lifetime distribution. Statistics 42(4): 351-362.
6. Morris J. S, Baggerly K. A. and Coomb.s K. R. 2003. Bayesian shrinkage estimation of the relative abundance of mRNS transcripts using SAGE. Biometrics 59: 476-486.
7. Chiou P. and Han C. P.1989. Shrinkage estimation of threshold parameter of the exponential distribution. IEEE transactions on reliability 38 (4): 449-453.
8. Singh H. P. and Raghuvanshi H. S. 1996. Some shrinkage estimators for the variance of exponential density. Microelectronic Reliability 36(5): 651-655.